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# The Anapole Moment of the Deuteron with the Argonne $v18$ Nucleon-Nucleon Interaction Model

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## Abstract

We calculate the deuteron anapole moment with the wave functions obtained from the Argonne  $v18$  nucleon-nucleon interaction model. The anapole moment operators are considered at the leading order. To minimize the uncertainty due to a lack of current conservation, we calculate the matrix element of the anapole moment from the original definition. In virtue of accurate wave functions, we can obtain a more precise value of the deuteron anapole moment which contains less uncertainty than the former works. We obtain a result reduced by more than 25% in the magnitude of the deuteron anapole moment. The reduction of individual nuclear contributions is much more important however, varying from a factor 2 for the spin part to a factor 4 for the convection and associated two-body currents.

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# 1 Introduction

After Zel'dovich introduced the concept of the anapole moment (AM) [1], a first non-zero measurement was reported quite recently in the  $^{133}\text{Cs}$  atom [2]. Isolation of the effect is difficult because the AM has a small contribution compared to the leading order  $Z^0$  exchange in atomic physics. However, the part that involves the nuclear spin is suppressed by a factor  $1 - 4\sin^2\theta_w^2$  ( $\simeq 0.08$ ). Thus in some cases, the AM which is a higher-order effect in electroweak interactions can be comparable with the spin-dependent  $Z^0$  contribution. Flambaum and Khriplovich showed that the AM of a heavy nucleus is proportional to  $A^{2/3}$  [3], from which one can deduce the dominance of the AM over  $Z^0$  exchange or radiative corrections for large  $A$ . Since then, calculations of the AM of heavy nuclei have been the object of the major interest in the theoretical works of the domain [3, 4, 5, 6, 7].

The dominant contribution to the AM of a light system, namely the deuteron, was pointed out by one of the author [8]. The spin current which stems from the anomalous magnetic moment of the nucleon was expected to give a few times larger contribution than the convection current or exchange currents and it was verified in several papers [9, 10, 11, 12]. As a result, the contribution of the AM becomes similar to the radiative corrections. Khriplovich and Korkin (KK in short) calculated the AM of the deuteron analytically with the zero-range-approximated (ZRA) wave functions [11]. Their result of the spin-current term, which is the most dominant contribution to the AM of the deuteron, has exactly the same form as the one obtained from the framework of the effective field theory by Savage and Springer (SS in short) [10]. The error of the result, which may be mainly from the simple wave functions, is estimated to be about 20%. However, in recent calculations of the asymmetry in the  $\vec{n} + p \rightarrow d + \gamma$ , we showed that the ZRA (or the effective field theory [13]) result exceeds the ones obtained with a few phenomenological wave functions by more than 50% [14, 15]. With this observation, it may be possible that the error of the deuteron AM with ZRA wave functions can be larger than 20%. With the purpose to minimize theoretical uncertainties, we calculate the deuteron AM with the wave functions obtained from the Argonne  $v18$  potentials [16].

The low momentum transfer which characterizes the process makes it possible to treat the problem with heavy-baryon-chiral-perturbation theory (HBChPT). With the counting rules of HBChPT, one can obtain the transition operators order-by-order in a well-defined way. The error of a calculation can be systematically estimated from the higher orders that are not taken into account in the calculation. Since well-ordered operators are evaluated with very accurate phenomenological wave functions, we expect that the uncertainties will be the least among the theoretical calculations.

## 2 The Anapole Moment

The anapole moment of a system is obtained from the expansion of the vector-potential

$$A_i(\mathbf{x}) = \int d\mathbf{x}' \frac{j_i(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \quad (1)$$

in a series in  $x^{-1}$  ( $x \equiv |\mathbf{x}|$ ). The quantity  $\mathbf{j}$  is the matrix element of the current density operator for given initial and final states

$$\mathbf{j} = \langle \psi_f | \hat{\mathbf{j}} | \psi_i \rangle. \quad (2)$$

The zeroth order term vanishes since there is no net current and the first-order term gives the vector potential of the magnetic dipole. The second order can be separated into a magnetic quadrupole term and the anapole term. After some algebra, one obtains the following form of the anapole term <sup>#3</sup>

$$\mathbf{a} \equiv \frac{2\pi}{3} \int d\mathbf{x} \mathbf{x} \times (\mathbf{x} \times \mathbf{j}(\mathbf{x})) \quad (3)$$

with which the vector potential of the anapole moment reads

$$\mathbf{A}_{\text{anapole}}(\mathbf{x}) = \frac{1}{4\pi} \left( -\mathbf{a} \nabla^2 \frac{1}{x} + \mathbf{a} \cdot \nabla \nabla \frac{1}{x} \right). \quad (4)$$

The second term of  $\mathbf{A}_{\text{anapole}}$  can be removed by a suitable choice of gauge and the resultant anapole vector potential takes the form

$$\mathbf{A}_{\text{anapole}}(\mathbf{x}) = \mathbf{a} \delta^{(3)}(\mathbf{x}). \quad (5)$$

If current conservation is satisfied, Eq. (3) can also be written as

$$\mathbf{a} = -\pi \int d\mathbf{x} x^2 \mathbf{j}(\mathbf{x}). \quad (6)$$

In many calculations of the nuclear AM, Eq. (6) was adopted as the working definition of the AM. However, as shown in [7], the contribution of a current term depends strongly on this definition while the total result may be sensitive on fulfilling the current-conservation constraint. The situation becomes more uncertain for the exchange currents or higher order terms. Thus in order to avoid possible large effect on the result due to lack of current conservation, we preferentially adopt in our calculations the original definition, Eq. (3). We nevertheless stress that we include in our work a minimal set of two-body currents ensuring current conservation in relation with the parity-non-conserving (PNC) one-pion exchange interaction. This one (together with the PC part) turns out to be essential in ensuring the approximate equivalence of Eq. (6) with Eq. (3), as will be seen at the end of the paper.

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<sup>#3</sup>While we have the AM in fm<sup>2</sup>, the Seattle group favors a dimensionless definition which can be obtained by multiplying ours by  $m_N^2/4\pi$ .

### 3 Operators and Wave Function

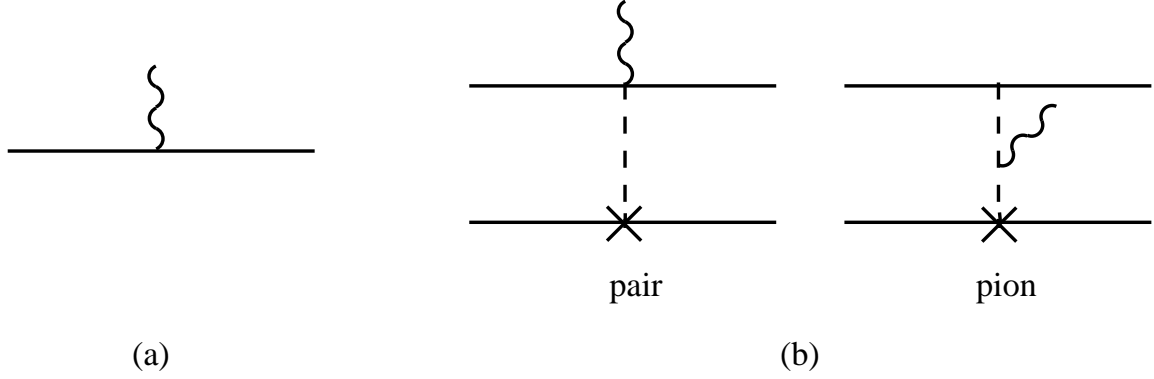


Figure 1: Diagrams representing one and two-body electromagnetic contributions.

The diagrams considered are shown in Fig. 1. Leading one-body currents (Fig. 1-(a)) are composed of spin and convection currents and PNC exchange contributions are in Fig. 1-(b). The leading PNC vertex marked with  $\times$  reads [17]

$$\mathcal{L}_{\pi NN}^{(1)} = -\frac{h_{\pi NN}^{(1)}}{\sqrt{2}} N^\dagger (\vec{\tau} \times \vec{\pi})^z N. \quad (7)$$

The current density operators of each diagram are

$$\hat{\mathbf{j}}_{\text{spin}}(\mathbf{x}) = e \sum_{i=1}^2 \frac{\mu_N^i}{2m_N} \nabla_{\mathbf{x}} \times (\vec{\sigma}_i \delta^{(3)}(\mathbf{x} - \mathbf{r}_i)), \quad (8)$$

$$\hat{\mathbf{j}}_{\text{conv}}(\mathbf{x}) = e \sum_{i=1}^2 \frac{1 + \tau_i^z}{4m_N} \{ \mathbf{p}_i, \delta^{(3)}(\mathbf{x} - \mathbf{r}_i) \}, \quad (9)$$

$$\hat{\mathbf{j}}_{\text{pair}}(\mathbf{x}) = -e \frac{g_A h_{\pi NN}^{(1)}}{2\sqrt{2}f_\pi} (\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^z \tau_2^z) y_0(r_{12}) \sum_{i=1}^2 \vec{\sigma}_i \delta^{(3)}(\mathbf{x} - \mathbf{r}_i), \quad (10)$$

$$\hat{\mathbf{j}}_{\text{pion}}(\mathbf{x}) = -e \frac{g_A h_{\pi NN}^{(1)}}{2\sqrt{2}f_\pi} (\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^z \tau_2^z) (\vec{\sigma}_1 \cdot \vec{\partial}_1 - \vec{\sigma}_2 \cdot \vec{\partial}_2) (\vec{\partial}_1 - \vec{\partial}_2) y_0(r_{1x}) y_0(r_{2x}) \quad (11)$$

where  $r_{12} \equiv |\mathbf{r}_1 - \mathbf{r}_2|$ ,  $r_{ix} \equiv |\mathbf{r}_i - \mathbf{x}|$  and

$$y_0(r) \equiv \frac{e^{-m_\pi r}}{4\pi r}.$$

$\mu_N^i$  is defined as

$$\mu_N^i \equiv \frac{1}{2} (\mu_S + \tau_i^z \mu_V)$$

with  $\mu_S = 0.88$  and  $\mu_V = 4.71$ . The PNC interaction of the proton and the neutron generates parity-odd components in the deuteron wave function. In the context of the meson-exchange

picture, the PNC interaction is mediated by  $\pi$ ,  $\rho$ ,  $\omega$  and heavier mesons. PNC components in the wave function can be obtained by solving the Schrödinger equation with the PNC potentials given in [17]. In a low energy process, it is believed that the pion exchange will dominate PNC interactions when its contribution is not forbidden by some selection rule. The pion-exchange PNC potential reads

$$V_{\text{pnc}}^{1\pi}(\mathbf{r}) = \frac{g_A h_{\pi NN}^{(1)}}{\sqrt{2} f_\pi} (\vec{\tau}_1 \times \vec{\tau}_2)^z \mathbf{I} \cdot \hat{\mathbf{r}} \frac{d}{dr} y_0(r) \quad (12)$$

where  $\mathbf{r} \equiv \mathbf{r}_p - \mathbf{r}_n$ ,  $r \equiv |\mathbf{r}|$  and  $\mathbf{I} \equiv \frac{1}{2}(\vec{\sigma}_p + \vec{\sigma}_n)$ . The constants,  $g_A$  and  $f_\pi$  are given the values 1.267 and 92.4 MeV respectively. For the deuteron, the PNC potential, Eq. (12) gives rise to a parity-odd  $^3P_1$  component. We write the parity admixed wave function as

$$\psi_d(\mathbf{r}) = \frac{1}{\sqrt{4\pi r}} \left[ \left( u(r) + S_{12}(\hat{\mathbf{r}}) \frac{w(r)}{\sqrt{8}} \right) \zeta_{00} - i h_{\pi NN}^{(1)} \sqrt{\frac{3}{2}} \mathbf{I} \cdot \hat{\mathbf{r}} v(r) \zeta_{10} \right] \chi_{1J_z} \quad (13)$$

where  $S_{12}(\hat{\mathbf{r}}) \equiv 3 \vec{\sigma}_1 \cdot \hat{\mathbf{r}} \vec{\sigma}_2 \cdot \hat{\mathbf{r}} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$  and  $\chi$  and  $\zeta$  represent a spinor and an isospinor, respectively. The quantities  $u(r)$ ,  $w(r)$  and  $v(r)$  are obtained by solving the Schrödinger equation and one can calculate the matrix elements of the current density operators, Eqs. (8) – (11), with the obtained solutions. After that, the calculation of the anapole moment with Eq. (3) is straightforward.

## 4 Results

The anapole moment, Eq. (3), reads for each term

$$\mathbf{a}_{\text{spin}} = -\mu_V \sqrt{\frac{1}{6}} \frac{\pi}{m_N} \int dr r v(r) \left( u(r) - \sqrt{2} w(r) \right) e \mathbf{I} h_{\pi NN}^{(1)}, \quad (14)$$

$$\mathbf{a}_{\text{conv}} = \frac{1}{3} \sqrt{\frac{1}{6}} \frac{\pi}{m_N} \int dr r v(r) \left( u(r) - \sqrt{2} w(r) \right) e \mathbf{I} h_{\pi NN}^{(1)}, \quad (15)$$

$$\mathbf{a}_{\text{pair}} = -\frac{\sqrt{2}\pi g_A}{9f_\pi} \int dr r^2 y_0(r) \left( u(r) + \frac{1}{\sqrt{2}} w(r) \right) \left( u(r) - \sqrt{2} w(r) \right) e \mathbf{I} h_{\pi NN}^{(1)}, \quad (16)$$

$$\mathbf{a}_{\text{pion}} = \frac{\sqrt{2}\pi g_A}{3f_\pi m_\pi} \int dr r y_0(r) \left( u(r) + \frac{1}{\sqrt{2}} w(r) \right) \times \left[ u(r) \left( 1 - \frac{1}{3} m_\pi r \right) - \frac{1}{\sqrt{2}} w(r) \left( 1 + \frac{1}{3} m_\pi r \right) \right] e \mathbf{I} h_{\pi NN}^{(1)}. \quad (17)$$

Numerical results are

$$\mathbf{a}_{\text{spin}} = -0.531 e \mathbf{I} h_{\pi NN}^{(1)}, \quad (18)$$

$$\mathbf{a}_{\text{conv}} = 0.038 e \mathbf{I} h_{\pi NN}^{(1)}, \quad (19)$$

$$\mathbf{a}_{\text{pair}} = -0.026 e \mathbf{I} h_{\pi NN}^{(1)}, \quad (20)$$

$$\mathbf{a}_{\text{pion}} = 0.027 e \mathbf{I} h_{\pi NN}^{(1)} \quad (21)$$

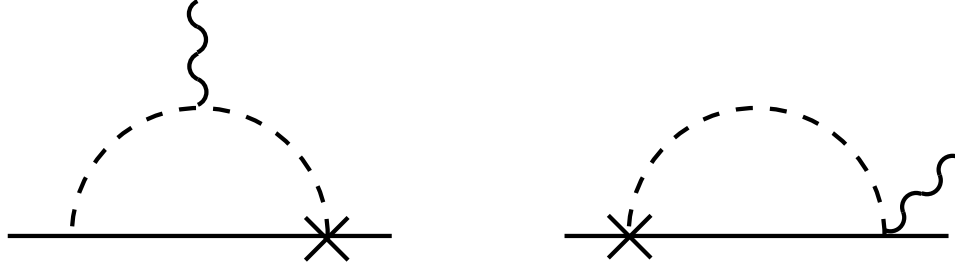


Figure 2: The diagrams of the leading order nucleonic anapole moment. Solid, dashed and wavy lines represent the nucleon, pion and photons, respectively.

where all the values are in the  $\text{fm}^2$  unit.

In the HBChPT, one can specify the magnitude of a diagram in terms of the power ( $\nu$ ) of the momentum transfer ( $Q$ ) by the rule

$$\nu = 2L - 2(C - 1) - 1 + \sum_i \nu_i \quad (22)$$

where

$$\nu_i = d_i + \frac{n_i}{2} + e_i - 2. \quad (23)$$

$L$  is the number of loops,  $C$  is the number of disconnected lines,  $d_i$ ,  $n_i$  and  $e_i$  are the numbers of derivatives, nucleon lines and external gauge fields at the vertex  $i$ , respectively. All the strong and electro-magnetic vertices satisfy

$$\nu_i \geq 0 \quad (24)$$

but the PNC vertex, Eq. (7) has  $\nu_i = -1$ . One can easily verify that the diagrams in Fig. 1 have  $\nu = -2$ . Nucleon anapole diagrams shown in Fig. 2 also have this order. In order to be consistent with the order counting of the effective field theory, the nucleonic terms also should be included.

This term was already calculated in several works [18, 19, 20, 21]. Since the results are consistent in both magnitude and sign, we do not repeat its calculation but just adopt the result here. The leading order nucleonic anapole, taking into account the deuteron D-state probability  $P_D$ , reads

$$\begin{aligned} \mathbf{a}_N &= -\frac{g_A}{6\sqrt{2}f_\pi m_\pi} \left(1 - \frac{3}{2}P_D\right) e \mathbf{I} h_{\pi NN}^{(1)} \\ &= -0.417 e \mathbf{I} h_{\pi NN}^{(1)}. \end{aligned} \quad (25)$$

Then the anapole moment of the deuteron at the leading order is

$$\begin{aligned}
\mathbf{a}_d &= \mathbf{a}_{\text{spin}} + \mathbf{a}_{\text{conv}} + \mathbf{a}_{\text{pair}} + \mathbf{a}_{\text{pion}} + \mathbf{a}_N \\
&= -(0.531 - 0.038 + 0.026 - 0.027 + 0.417) e \mathbf{I} h_{\pi NN}^{(1)} \\
&= -0.909 e \mathbf{I} h_{\pi NN}^{(1)}.
\end{aligned} \tag{26}$$

## 5 Discussions

As expected, the spin term is the most dominant one among the contributions from the currents. The contributions from the convection, pair and pion terms are much smaller while the two last ones strongly cancel. The sum of these three contributions is only about 7% of the spin term. However the nucleonic term is close to the spin term, which is quite contradictory to the former results of the deuteron anapole moment.

In order to illustrate this feature clearly, let us compare our result with the ones by SS [10] and KK [11]. Firstly, SS's result is

$$\begin{aligned}
\mathbf{a}_d^{\text{SS}} &= \mathbf{a}_{\text{spin}}^{\text{SS}} + \mathbf{a}_{\text{PE}}^{\text{SS}} + \mathbf{a}_N \\
&= -(1.03 - 0.18 + 0.46) e \mathbf{I} h_{\pi NN}^{(1)} = -1.31 e \mathbf{I} h_{\pi NN}^{(1)},
\end{aligned} \tag{27}$$

where the pion-exchange term (PE) corresponds to the sum of the convection, pair and pion terms in our calculation. We have verified in a separate calculation that SS's result can be obtained with the ZRA wave function [22]. Secondly, KK's result reads

$$\begin{aligned}
\mathbf{a}_d^{\text{KK}} &= \mathbf{a}_{\text{spin}}^{\text{KK}} + \mathbf{a}_{\text{orb}}^{\text{KK}} + \mathbf{a}_N^{\text{KK}} \\
&= -(1.03 - 0.07 + 0.21) e \mathbf{I} h_{\pi NN}^{(1)} = -1.17 e \mathbf{I} h_{\pi NN}^{(1)}.
\end{aligned} \tag{28}$$

Since KK also used the ZRA wave function, their spin term is equal to SS's one. However there are substantial differences in the remaining terms. In the following, we successively discuss these different contributions.

The biggest quantitative difference of our result with the above ones comes from the spin term. The *Av18* result is roughly half of the ZRA value. In [11], KK argued that the uncertainty of the ZRA wave function is about 20%. Taking this uncertainty into consideration, the spin term can be reduced to  $-0.82 e \mathbf{I} h_{\pi NN}^{(1)}$  which is still larger than our result by about 55%. The discrepancy of our result with the KK (as well as SS) estimate is understood by noting that our calculation incorporates the effect of a short-range repulsion in S-states as well as the known repulsive character of the NN interaction in the  $^3P_1$  channel. We also note that the ZRA wave function contains only a central component,  $u(r)$ , while our calculation includes a tensor one,  $w(r)$  too. Contrary to the PNC asymmetry in n-p radiative capture



[14, 15], this component produces a partly destructive interference. The contribution of each part is

$$\mathbf{a}_{\text{spin}}^{\text{cen}} = -0.724 e \mathbf{I} h_{\pi NN}^{(1)} \quad (29)$$

$$\mathbf{a}_{\text{spin}}^{\text{ten}} = 0.193 e \mathbf{I} h_{\pi NN}^{(1)}. \quad (30)$$

In some cases, the contribution of the tensor part in the wave function is assumed to be small and its contribution is neglected. However in the deuteron anapole moment, this contribution is non-negligible and its effect on the magnitude of the anapole moment is substantial.

We now turn to the second contribution in Eqs. (27, 28), which, among other contributions, involves the convection current one. There too, a large suppression of our results is observed but the effect is more drastic for the PE term in SS's result than for the orbital one in KK's result. In SS's result, PE term's contribution is about 15% of the total magnitude, but in our result it is only 4% of the total value. Then, the question arises why the PE term in SS does not coincide with the orbital term in KK.

It can be shown that the anapole moment of the convection term with the definition Eq. (3) is equivalent to the orbital term

$$\mathbf{a}_{\text{conv}} = \left\langle -i \frac{\pi e}{12m_N} [\mathbf{l}^2, \mathbf{r}] \right\rangle \quad (31)$$

where  $\mathbf{l} \equiv \mathbf{r} \times \mathbf{p}$  is the angular momentum operator in the center of mass frame. The global factor of the sum of the spin and the convection term,  $(\mu_V - 1/3)$  always appears regardless of wave functions. This explains the relative ratio of the contributions due to the spin and convection currents in KK's work as well as in ours.

As mentioned above, KK's orbital term accounts for only the convection term. In KK's argument, they made use of the fact that

$$\mathbf{J}_{\text{pair}}(\mathbf{r}) + \mathbf{J}_{\text{pion}}(\mathbf{r}) \propto \mathbf{r}. \quad (32)$$

With the definition Eq. (3), it may be that this term gives a zero contribution since

$$\mathbf{a} \propto \mathbf{r} \times (\mathbf{r} \times \mathbf{J}). \quad (33)$$

We would like to point out that caution is required in the above reasoning. Firstly, the matrix elements that are relevant in the calculation of the anapole moment involve the *current density* operator and not the *current* operator. It can be easily shown that the current operator which is obtained by integrating the current density operator with respect to the field point  $\mathbf{x}$  does satisfy Eq. (32). However the anapole moment should be obtained from the double vector product of the current density operator and field point  $\mathbf{x}$ , whose result may not be like Eq. (32) in general. Secondly, a rough substitution  $\mathbf{x} \rightarrow \mathbf{r}$  in Eq. (3)

gives Eq. (33). This substitution, even if one is working in the center of mass frame and thus center of mass coordinate is discarded, should be carefully derived from the evaluation of the matrix elements with the two-body wave function. A crude transformation of the coordinates may give wrong results. As our result shows, a rigorous derivation with the *current density* operator gives a non-zero contribution of the two-body currents. We also checked that this derivation provides coincidence with SS's result [22]. Curiously, the extra contributions that the SS's work accounts for do not seem to show up in our results. It turns out that the term proportional to  $1/m_\pi$  in Eq. (21), a priori favored, is suppressed with the ZRA value (numerical factor 0.07 instead of 0.18). It is reminded that in the zero-pion-mass limit, this term is cancelled by another one arising from the nucleonic term [10]. The other term (Eq. (20) and part of Eq. (21)), which is less singular in configuration space and has an opposite sign, is essentially unchanged. As a result, the sum of the two contributions almost vanishes in our work.

Concerning the nucleonic term (third one in Eqs. (27, 28)), KK adopt

$$\mathbf{a}_N^{\text{KK}} = -\frac{g_A}{6\sqrt{2}f_\pi m_\pi} \left(1 - \frac{6m_\pi}{\pi m_N} \ln \frac{m_N}{m_\pi}\right) e \mathbf{I} h_{\pi NN}^{(1)}. \quad (34)$$

The first term in the parenthesis coincides with the one we retained and the second term stems from the  $1/m_N$  correction to the first term. In the context of the counting rule of the effective field theory, this  $1/m_N$  term is classified in the higher order corrections to the leading term. In order to be consistent with the strategy of the effective field theory, the  $1/m_N$  corrections should be taken into account consistently, i.e. their correction should be calculated not only for the nucleonic term but also for current terms or any other types that have the same order. In that sense, KK's result contains the  $1/m_N$  correction partially. However, it is interesting to notice that the magnitude of the  $1/m_N$  correction to the nucleonic term amounts to a half of its leading value. This indicates that the higher order corrections can play a critical role in the magnitude of the deuteron anapole moment. Their calculation should be done in the future.

In a separate work, we have observed that, while the spin current satisfies gauge invariance by itself, convection, pair and pion currents are not gauge invariant solely and gauge invariance is restored when the three terms are summed up in a minimal case [22]. In the general case, one should also include two-body currents related to the description of the NN strong interaction. As a way to investigate how much gauge invariance is broken, we compare the anapole moments calculated with Eq. (3) and Eq. (6). Equation (3) is defined from the definition of the anapole vector potential and Eq. (6) is derived from Eq. (3) by imposing current conservation. Calculation of the anapole moment from Eq. (6) is straightforward too. The results are

$$\mathbf{a}_{\text{conv}}^{\text{CC}} = 0.051 e \mathbf{I} h_{\pi NN}^{(1)}, \quad (35)$$

$$\mathbf{a}_{\text{pair}}^{\text{CC}} = -0.076 e \mathbf{I} h_{\pi NN}^{(1)}, \quad (36)$$

$$\mathbf{a}_{\text{pion}}^{\text{CC}} = 0.045 e \mathbf{I} h_{\pi NN}^{(1)}. \quad (37)$$

where the superscript CC implies the result with the assumption of current conservation. As expected, individual contributions differ from those in Eqs. (19-21), but it is also noticed that the sum differs,  $0.020 e \mathbf{I} h_{\pi NN}^{(1)}$ , instead of  $0.039 e \mathbf{I} h_{\pi NN}^{(1)}$ . The first candidate to explain the discrepancy is the two-body current associated to the strong one-pion exchange interaction. The corresponding contribution,  $0.018 e \mathbf{I} h_{\pi NN}^{(1)}$ , which should be added to the sum of contributions of Eqs. (35-37), removes most of the difference. This one-pion exchange interaction can also generate self-gauge invariant contributions that will affect equally the sum of contributions in Eqs. (19-21) and Eqs. (35-37). Due to cancellations of pair and pion contributions, this common contribution is found to be small ( $-0.0007 e \mathbf{I} h_{\pi NN}^{(1)}$ ).

Concluding, we calculated the anapole moment of the deuteron with the wave functions obtained from the *Av18* potential. Its magnitude is reduced by more than 25% from the previous ZRA results but is still comparable to the radiative corrections. If the contribution of the nucleon anapole moment is put aside, the effect is much larger however, ranging from a factor 2 for the spin contribution to a factor 4 for the contribution of the convection current and associated two-body currents. We observed that gauge invariance for these last contributions, hopefully smaller, is a severe constraint and its fulfillment is a non-trivial problem. In the analysis of other's work, we noticed that the corrections of higher orders can modify significantly their estimate. The calculation of gauge-invariant higher-order corrections will be the future challenge.

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